First graders' work on additive problems with the use of different notational tools

El trabajo de alumnos de primer grado en problemas aditivos con el uso de diferentes herramientas notacionales

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Abstract

We explored the use of three types of notational tools (paper and pencil, unlabeled tables, and labeled tables) with twenty-two children in Grade 1 who were solving Vergnaud's Category II and IV additive problems. Our results indicate that for Category II problems, which are simpler, children have a higher rate of success when they are able to solve them orally, without any notational tool. However, for Category IV problems, which are more complex, children perform better when they have access to notational tools than when they solve the problems without access to notations. Among the children interviewed for this study, unlabeled tables help them the most, followed by the use of paper and pencil.

Keywords: Multi-literacies, notational tools, additive problems, tables.

Resumen

Exploramos el uso de tres diferentes herramientas notacionales (papel y lápiz, tablas sin etiquetas y tablas previamente etiquetadas) con veintidós niños de primer grado que resolvieron problemas aditivos de las Categorías II y IV de acuerdo con la clasificación de Vergnaud. Nuestros resultados indican que para los problemas de la Categoría II, más simples, los niños obtuvieron mejores resultados cuando los resolvieron en forma exclusivamente oral, sin herramientas notacionales. Sin embargo, para los problemas de la Categoría IV, que son más complejos, los niños presentaron una mejor performance cuando utilizaron las herramientas notacionales. Las herramientas más útiles, para los niños entrevistados en este estudio, fueron las tablas sin etiquetas seguidas del papel y lápiz.

Palabras clave: Multi-letrismo, herramientas notacionales, problemas aditivos, tablas.

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Introduction

Vergnaud's now seminal classification of problems in the conceptual field of additive structures (1982), as well as his description of children's ability to deal with these different kinds of problems, sets the stage for the study described in this paper. Vergnaud (1982) described six basic categories of additive relationships among quantities. In his 1982 paper, Vergnaud reports on a study by Vergnaud and Durand in which they examined differences between two categories of problems: A transformation between two measures (*state, transformation, state* or Category II) and composition of two transformations (*transformation, transformation, transformation* or Category IV; see Table 1). These are the same categories of problems that we focused on in our study. Vergnaud and Durand found that Category II problems lead to higher numbers of correct responses than Category IV problems; that is, they found a clear gap between adding two transformations (Category IV) and applying a transformation to a state (Category II).

Category II problems involve a transformation linking two measures. As shown in Figure 1, in Category II type problems one can:

- find c, knowing a and b
- find b, knowing a and c
- find a, knowing b and c

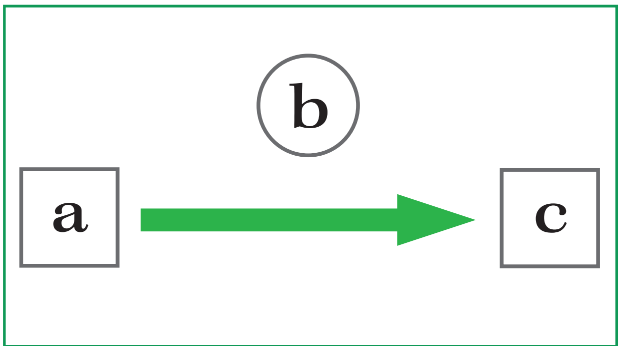

Fig. 1: Vergnaud's Category II problems (Vergnaud, 1982). The horizontal arrow indicates a transformation linking a measure to another one.

Category IV problems involve a composition of two transformations. As shown in Figure 2, in Category IV type problems one can:

- find c, knowing a and b
- find b, knowing a and c
- find a, knowing b and c

In the exploratory study reported in this paper, we were interested in investigating the impact of the use of different notations on children's abilities to solve Vergnaud's Category II and IV additive problems. The research questions underlying this study are: *Does the use of notations (specifically, paper and pencil, unlabeled tables, and labeled tables) aid children's solution of Category II and IV additive problems? Are there notations that are particularly useful for children?* This paper reports on this exploratory study, which intends to lay the ground for future research looking at the impact of the use of different notational tools while solving additive problems.

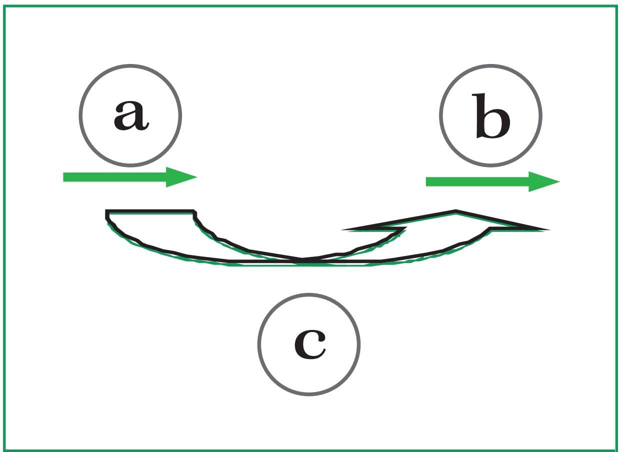

Fig. 2: Vergnaud's Category IV problems (Vergnaud, 1982). The lower arrow symbolizes the composition of the two upper ones. The states are unknown.

In this paper, we will use the term *notations* to refer to what some others have called external representations (see Goldin, 1998; Martí & Pozo, 2000), to differentiate from mental representations. The main features of these external representations are that they are made with pencil and paper and have a physical existence.

In his 1982 paper, Vergnaud does not describe providing young children with or requesting young children to produce particular notations for solving problems. However, he does propose two criteria for the efficiency of "symbolic representations" (p. 53). The first is "symbolic representations should help students to solve problems that they would otherwise fail to solve" and the second is that "symbolic representations should help students in differentiating various structures and classes of problems" (p. 53). Vergnaud also highlights that "these criteria should be used to evaluate different sorts of symbolic systems, at different stages of the acquisition of additive structures" (p. 53). His proposal is that we should take the time to examine different symbolic systems and see what they can symbolize correctly, what limits they have, and what their advantages and inconveniences are. Our research questions in this study relate directly to Vergnaud's proposal.

Even though in Vergnaud's work there is no description of the use of notations with young children, Vergnaud describes an experiment with 11- to 13-year-old children on a variety of additive problems. The results of the experiment he describes leads Vergnaud to argue that diagrams may be more appropriate at this age (11-13 years of age) than equations for the solution of problems. In our study, we only worked with young children (i.e., while we focused on Grade 1, or ages 6-7, Vergnaud's experiment that focused on the use of notations focused on 11 to 13-year-old children) and we do not include an intervention component while Vergnaud's study did include an intervention; but, as Vergnaud, we want to be able to associate different notations with different problems.

Studies that look at the impact of the use of notations

Vasconcelos (1998) developed a study that involved the use of some of Vergnaud's additive problems, in didactical situations, and included the use of different kinds of notations. Vasconcelos had 3 groups of 8-year-old students in her study. Each group was provided with a pre-test, a teaching intervention, and a post-test. Each of the three groups received a training that consisted in the solving of addition and subtraction problems using three different tools: one group used Vergnaud's diagrams (1982), another Riley, Greeno, and Heller's (1983) part-whole diagrams, and a third group used manipulative materials.

In her study, Vasconcelos found that while all three groups showed increases in the number of correct responses from the pre to the post-test, it was those children using Vergnaud's diagrams who were able to achieve the greatest increases in the number of correct responses, and the group that used manipulative materials achieved the least increase in the number of correct responses. Vasconcelos found that the part-whole diagrams were not helpful in all cases. She thus concludes that the three different tools that she used in her research were not equally effective. Vasconcelos' findings resonate with Vergnaud's point about the usefulness of “symbolic representations.” The study we describe in this paper differs from Vasconcelos' in that we did not include an intervention, and in the fact that we focused on the use of notations that are part and parcel of standard or conventional mathematics and daily life practices: tables.

The study described in this paper connects to a larger line of research that explores the ways in which notations transform cognition, and are not merely external aides (e.g., Olson, 1994; Ong, 1982; Goody, 1977; Zhang & Norman, 1995). In their research, Zhang and Norman (1995) have explored not only the impact of different representations (this is their use of the term), but also the *differential impact* of different representations. From these researchers' perspective, each notation has a differential impact on its users (each notation even has different impacts across different users), referring to this as a “representational effect.” As they state, “different representations of a common abstract structure can cause dramatically different cognitive behaviors” (p. 271).

The literature in mathematics education has, in general, not explored children's use of tables. Exceptions to this pattern are found in the work of Brizuela and Lara Roth (2002) and Martí (2009). In their research, Brizuela and Lara-Roth (2002) showed that very young students (7-year-olds) could use tables to solve problems of an algebraic nature. Even though the students they interviewed had not received direct instruction in the use and setting up of tables, they were still able to use the tables to work through a problem. The tables in Brizuela and Lara-Roth's study were self-generated, and had no imposed structure to them; this is a fundamental difference with the kinds of tables used in the study described in this paper, in which both labeled and unlabeled tables, both including a grid, were used with children. Martí (2009) also explored the use of tables among young students, finding that, “the process of table construction can change the subject's prior knowledge” (p. 12). Martí clearly connects his research to studies that have explored the impact of different notational tools. In this paper, we will explore the differential impact of paper and pencil, labeled tables, and unlabeled tables.

Methodology

Participants

Twenty-two first grade children attending a public school in an urban suburb of Boston, MA, USA were interviewed individually during the first semester in the school year. Children need to be 6 years of age at the time of entering first grade. In their schooling, children had not been exposed to Category IV problems. In terms of Category II problems, children were not familiar with problems that did not end with an unknown. In fact, one child stated in one of the interviews, “This is just like math, but with no numbers in the beginning or in the end!” In the mathematics curriculum these children were exposed to, tables were not used as tools to solve problems, although they could be exposed to them as a way to display data.

Procedure

Children were randomly assigned to one of three conditions: (1) *paper and pencil*: children were given a pencil and a blank sheet of paper; (2) *unlabeled table:* children were given a pencil and a piece of paper with an unlabeled table (basically, an empty grid); and (3) *labeled table*: children were given a pencil and a piece of paper with a labeled table (labels for the rows read: “Start,” “First Round,” “Second Round,” “End”). The notational supports were only provided for the last four problems presented to them. Eight of the 22 children were assigned to the *paper and pencil* condition; 7 children were assigned to the *unlabeled table* condition; and the other 7 children

Order of presentation	Category (Vergnaud, 1982)	Description	Structure of problem
1	II (oral—no pencil and paper)	Pedro has 6 marbles. He plays one round of marbles and loses 4 marbles. How many marbles does he have at the end of the game?	6-4=(2)
2	IV (oral—no pencil and paper)	Pablo plays two rounds in a game of marbles. In the first round he wins 5 marbles. In the second round, he loses 3 marbles. What happened in the game?	x+5-3=x(+2)
3	II	Bernardo plays marbles. He loses 7 marbles. At the end of the game, he has 3 marbles. How many marbles did he have at the beginning of the game?	(10)-7=3
4	II	Claudio has 5 marbles. He plays a round and after he finishes playing he has 9 marbles. What happened during the game?	5(+4)=9
5	IV	Cristian plays two rounds of marbles. In the first round he wins 5 marbles. Then he plays a second round. At the end of the game he has won 9 marbles. What happened during the second round?	x+5(+4)=x+9
6	IV	Bruno plays two rounds of marbles. He plays the first round, and then after the second round he loses 7 marbles. After the two rounds, he has won 3 marbles in total. What happened during the first round of the game?	x(+10)-7=x+3

Table 1: Problems presented to children in this study.
Terms between parentheses are the responses requested of the children.

were assigned to the *labeled table* condition.

Each child was presented with a total of six problems (see Table 1). First they were presented with one Category II and one Category IV type problem to solve orally, as a test and baseline in order to understand what the child could do without the support of the notations. After these two initial test problems, each child was presented with two Category II problems and two Category IV problems, as well as with the notational supports for the condition to which they had been assigned. Each step in the problem was read to each child. After each step, children were asked, “Could you show this on your paper/on your table?”. So, for instance, in Problem 3 in Table 1, children were told, “Bernardo plays marbles. He loses 7 marbles. Could you show this on paper/on your table? [Wait for children to show something on paper/on the table]. At the end of the game, he has 3 marbles. Could you show this on paper/on your table? [Wait for children to show something on paper/on the table.] How many marbles did he have at the beginning of the game? Could you show this on paper/on your table? [Wait for children to show something on paper/on the table]”.

Results

Children's responses were coded as correct if they provided correct numerical responses and also the correct operation involved in the problem. This was particularly important for Category IV problems, in which the children needed to provide not only the correct numerical response but also the correct transformation that was involved. For instance, in Problem 5 from Table 1 the correct numerical response is 4, but the correct numerical response and transformation would be “he *won* 4.” We only coded as correct responses that included the “won 4” option, in this case. Some children said “he had 4,” and these responses were not coded as correct. Table 2 displays the results obtained from children's responses to the problems presented to them in the interviews.

Group		Category II	Category IV
Paper and pencil	oral presentation	62.50%	12.50%
	paper and pencil	50.00%	31.25%
Unlabeled table	oral presentation	85.72%	0%
	unlabeled table	64.29%	50.00%
Labeled table	oral presentation	71.43%	28.57%
	labeled table	28.57%	0%

Table 2: Percentage of correct responses by problem category and condition type.

Category II problems

Children had a higher percentage of correct responses in Category II problems than in Category IV problems, consistent with Vergnaud's (1982) findings. Moreover, for Category II problems, children had a higher percentage of correct responses in the oral situation than in the situations in which they had some sort of notational support. For the problems for which children had some sort of notational support, children had a higher percentage of correct responses with the unlabeled table, followed by the paper and pencil condition, and finally the labeled table condition. One hypothesis for the differences between labeled and unlabeled tables is that in the first semester of first grade children's reading and writing skills are still quite basic, making the use of the labels on the labeled table unhelpful and distracting.

Category IV problems

The pattern of children's responses on Category IV problems was different for labeled tables versus unlabeled tables on one hand, and for paper and pencil on the other. As indicated above, labeled tables may have been too challenging for children in the first semester of first grade given the requirements for reading and writing in these tables. For the paper and pencil and unlabeled tables conditions, children had a higher percentage of correct responses when they used these notational supports versus none (i.e., the oral condition) and of these two supports, they had a higher percentage of correct responses when using unlabeled tables versus just paper and pencil.

Example: Gabriela, paper and pencil condition

Gabriela is one of the first grade students interviewed in the *paper and pencil* condition. Her responses illustrate children's spontaneous development of notations that contribute to problem solutions.

Figure 3 shows Gabriela's spontaneous notation for Bernardo's problem (Problem 3 from Table 1), the

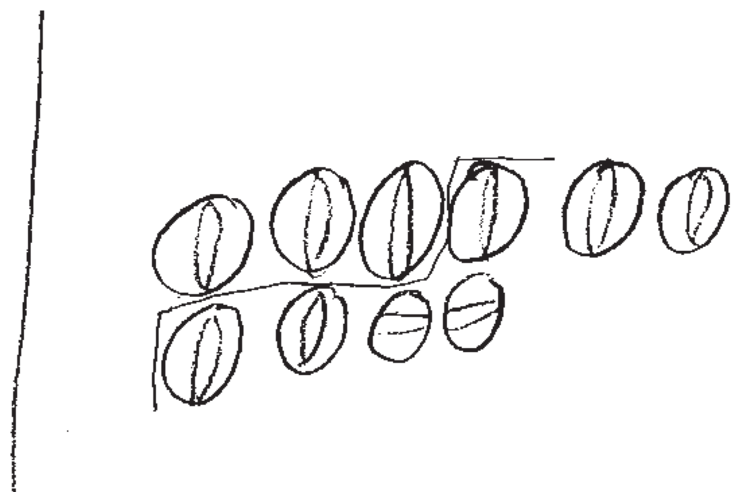

Fig. 3. Gabriela's notation for Bernardo's problem (Problem 3 from Table 1).

first for which she is asked to use pencil and paper. Her first inclination is to divide her piece of paper with a line down the paper (a bit of the line can be seen on the left side of Figure 3) and to draw quite detailed marbles. She first shows 7 marbles (the ones Bernardo lost), and then 3 marbles (the ones he ended up with at the end of the game), separating the group of 3 from the group of 7 by a line. When asked, “How many marbles did he have at the beginning of the game?”, Gabriela counts all the marbles she drew on her piece of paper and declares “ten.” When presented with the next Category II problem (Problem 4 from Table 1), Gabriela's notational strategy is very similar (see Figure 4): she draws marbles and separates the different quantities involved in the problem through the use of lines.

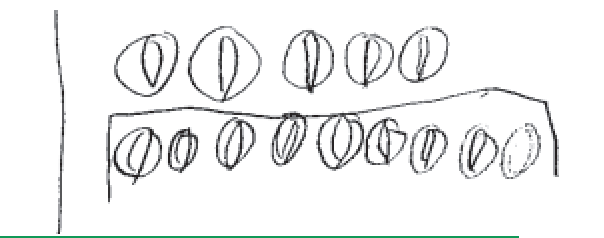

Fig. 4: Gabriela's notation for Claudio's problem (Problem 4 from Table 1).

However, Gabriela changes her notational strategy when presented with the first Category IV problem (see Figure 5).

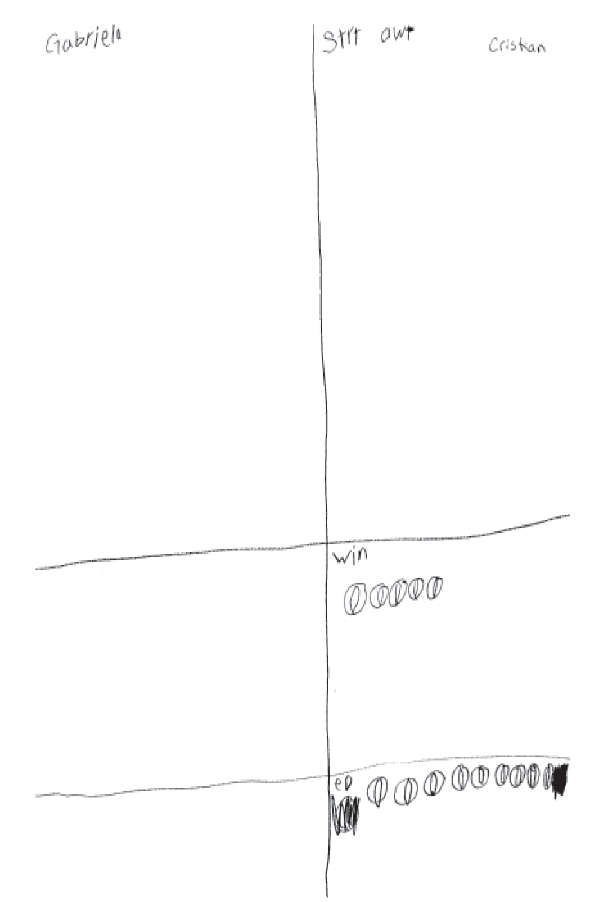

Fig. 5: Gabriela's notation for Cristian's problem (Problem 5 from Table 1).

Even before being read the problem, she first organizes her piece of paper, and sets up what we consider to be a grid or table: one longitudinal line down the middle of the paper and two horizontal lines, to set up two columns and three rows. She tells the interviewer that she will write the words “short”: on the top right column she writes “Strt OWT” (Start Out), below in the next row she writes “Win” and in the lower row she writes “eD” (End). We hypothesize that Gabriela wanted to make very clear to the interviewer that she was *purposefully* omitting letters. As a first grader, it is likely that she wanted to assert her literacy skills. Then, as the problem was read to her part by part, Gabriela proceeded to “complete her table.” When the interviewer stated that Cristian “wins 5 marbles,” Gabriela drew 5 marbles in her “Win” cell. Next, when the interviewer stated, “At the end of the game he has won 9 marbles,” Gabriela drew 9 marbles in the “eD” cell. To the final question she accurately responded that in the second round of the game Cristian “won 4 marbles” by carrying out a one to one correspondence between the 5 marbles in the “Win” cell and the first 5 marbles in the “eD” and then counting the extra marbles in the last row: 4 marbles.

Gabriela's example highlights the spontaneous way in which children naturally look to organize the data they are asked to deal with. Even though she was only provided with a blank piece of paper and a pencil, Gabriela looked to organize the space graphically, in much the same way we had planned for the children in the *labeled table* condition. We hypothesize that having realized the general structure of the problems, after having worked through four of the interview problems, she spontaneously decided to organize her piece of paper graphically and at the same time anticipated the kinds of information that she would need to keep track of in her piece of paper by labelling some of the cells in her table. We also hypothesize that her “short writing” captures the fact that she wants to be synthetic and efficient in her solving of these problems. In her notation for the last interview problem, she makes her writing even shorter: “St” for Start, “eD” for End, and “Wn” for Win (see Figure 6).

In this last problem, she adds a label for one of her empty cells while the problem is being read to her: “Lt” for Lost, not having anticipated that there would be a loss involved in the marbles game.

Discussion

Our results indicate a clear impact of the use of notations, particularly for the more complex Category IV problems. While children were able to perform quite well without notations when presented with Category II problems, with Category IV problems they faced many more challenges, for which notational supports in the form of either paper and pencil or unlabeled tables were particularly helpful. For Category II problems, for which oral presentations were most helpful for children, notational supports

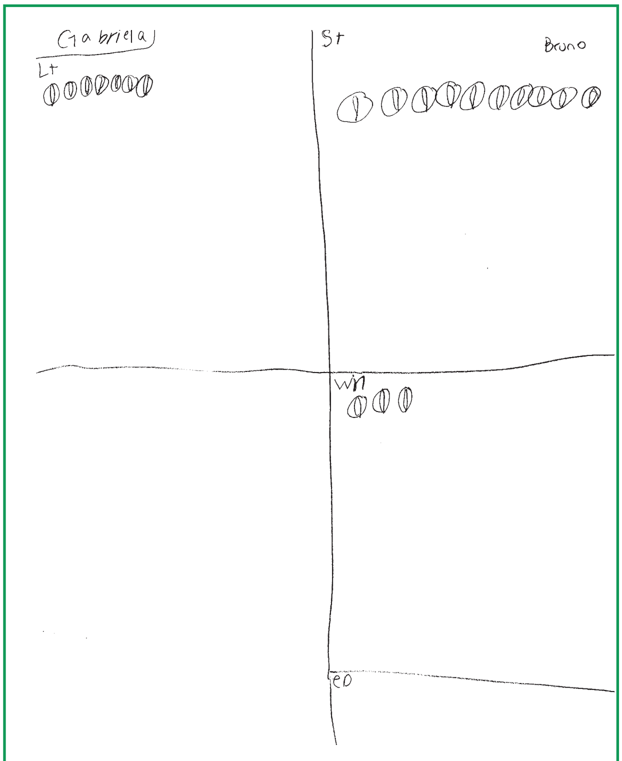

Fig. 6: Gabriela's notation for Bruno's problem (Problem 6 from Table 1).

were not uniformly helpful. The unlabeled tables were most helpful, followed by paper and pencil, and finally the labeled tables. In the case of the Category IV problems, the story we can tell is quite different. For these more complex problems, notational supports do in fact help and are more helpful than only oral presentations of problems. Even though the rate of correct responses is still lower for these problems compared to Category II problems, the opportunity to make notations gives the children we interviewed clear advantages.

Our study highlights the fact that we really cannot make blanket statements about what children can and cannot do. As in many other realms of human experience and learning, “it depends”. In the case of the conditions we explored in this study, how children perform on problems of differing complexity depends on the kinds of supports provided to them to solve the problems. Based on an analysis of the percentage of correct responses, our results indicate that there was an interaction between the type of notation used and the category of problem solved. When more complex problems are involved, children may perform better when they have access to notational tools than when they solve the problems without access to notations. These results resonate with Vergnaud's (1982) points regarding symbolism. Furthermore, different notations are more or less helpful for different types of problems, thus also connecting to Zhang and Norman's (1995) representational effect. Finally, given the increased strength of the mathematical work that can be accomplished when children have access to these notational tools, tables should become part and parcel of every mathematics and science lesson and classroom and should provide children with the opportunity to organize the data they interact with in meaningful ways.

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